

NEW SENIOR MATHEMATICS

ADVANCED
FOR YEARS 11 & 12
THIRD EDITION

J.B. FITZPATRICK
BOB AUS

NSW
STAGE 6

INTRODUCTION AND DEDICATION

J.B. Fitzpatrick

It is interesting to wonder whether J. B. Fitzpatrick ('Bernie') realised in 1983 just how popular his book *New Senior Mathematics* would be. That first edition of *New Senior Mathematics* was to remain in print for almost 30 years. It has stood the test of time thanks to the quality, rigour and variety of its questions, its accuracy and its high mathematical standards.

As Fitzpatrick wrote in 1983: 'Mathematics, like many other things, is best learnt by doing. A student begins to appreciate the power of mathematics when he or she has achieved a mastery of basic techniques, not after reading lengthy explanations ... The emphasis throughout the book is on the understanding of mathematical concepts.' (Introduction, *New Senior Mathematics* 1984.)

J. B. Fitzpatrick passed away in 2008. Fitzpatrick was a respected author, teacher and figurehead of mathematics education.

Bob Aus

Bob Aus taught in New South Wales high schools for 40 years, retiring in 2007. During that time Bob taught all courses from years 7 to 12 up to Level 1 / 4-unit / Extension 2. He has marked HSC examination papers and has been involved in the Standards Setting Process as Judge and Chief Judge for the three calculus-based courses over four years. He has also completed review work for the NSW Board of Studies and represented NSW at a week-long review and standards setting of the upper level course from each state prior to the development of the Australian National Curriculum for senior students.

Bob spent time as Regional Vocational Education Consultant in the North Coast Region and Mathematics Consultant in the Hunter Region. When he retired he was Head Teacher Mathematics at Merewether High School and enjoyed teaching an Extension 2 class with 24 students.

Bob's first publication was in 1983 and he has been involved with writing a range of textbooks and study guides since then, including revising and updating the *New Senior Mathematics* series 2nd Edition in 2013.

Bob has presented talks on the three calculus-based courses throughout the state. He has co-written the Years 6–9 Mathematics Syllabus for the Abu Dhabi Education Authority, as well as managing the writing project for support material for this course. He also wrote the years 10–12 syllabus for their calculus-based course.

This third edition of *New Senior Mathematics* updates the series for the new Stage 6 courses in New South Wales to be implemented in Year 11, 2019.

NEWSENIOR MATHEMATICS

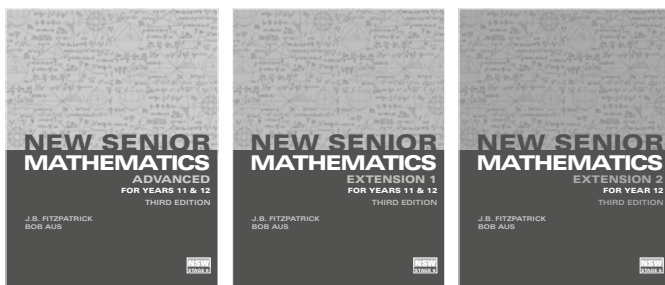
THIRD EDITION

New Senior Mathematics Advanced for Years 11 & 12 is part of a new edition of the well-known mathematics series for New South Wales. The series has been updated to address all requirements of the new Stage 6 syllabus. We have maintained our focus on mathematical rigour and challenging student questions, while providing new opportunities for students to consolidate their understanding of concepts and ideas with the aid of digital resources and activities.

Student Book

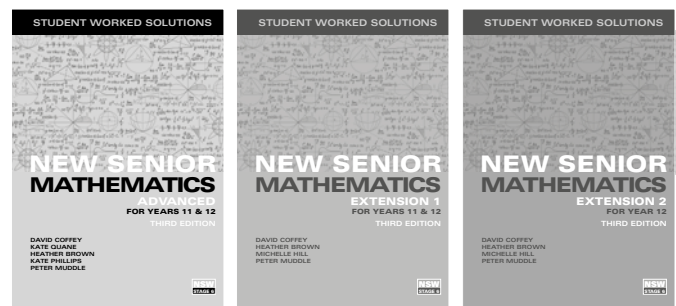
The first three chapters of the student book contain revision material that provides the necessary foundation for the development of senior mathematics concepts. In the new edition you'll also find:

- Content built on a rigorous, academic approach that promotes excellence and prepares students for higher education
- A simple convenient approach with Year 11 and 12 content in one book for Advanced and Extension
- Digital technology activities that promote deeper understanding allowing students to make connections, and visualise and manipulate data in real time.



Student Worked Solutions

The *New Senior Mathematics for Years 11 & 12 Student Worked Solutions* contain the fully worked solutions for every second question in *New Senior Mathematics for Years 11 & 12*.



Reader+

Reader+, our next generation eBook, features content and digital activities, with technology such as graphing software and spreadsheets, to help students engage on their devices.

Teacher Support includes materials such as practice exams, question banks, investigation assignments, and fully worked solutions to cover all internal and external assessment items to save you time.



FEATURES OF THE THIRD EDITION STUDENT BOOK/READER⁺

YEAR LEVELS

Year levels are indicated on each page for easy identification of Year 11 & 12 content.

YEAR 11

YEAR 12

MAKING CONNECTIONS

This eBook feature provides teachers and students with a visual interactive of specific mathematics concepts or ideas to aid students in their conceptual understanding.

MAKING CONNECTIONS

EXPLORE FURTHER

This eBook feature provides an opportunity for students to consolidate their understanding of concepts and ideas with the aid of technology, and answer a small number of questions to deepen their understanding and broaden their skill base. These activities should take approximately 5–15 minutes to complete.

EXPLORE FURTHER

CHAPTER REVIEW

Each chapter contains a comprehensive review of chapter content.

CHAPTER REVIEW

SUMMARY PAGES

A comprehensive course summary is provided at the end of the book.

SUMMARY

CONTENTS

Introduction and dedication iii

YEAR 11

| | | SYLLABUS REFERENCE |
|---|-----------|-----------------------|
| CHAPTER 1 Algebraic techniques | 1 | MA-F1.1 |
| 1.1 Simplifying algebraic expressions | 1 | |
| 1.2 Substitution in formulae | 2 | |
| 1.3 Basic polynomials | 3 | |
| 1.4 Factorising by grouping in pairs | 4 | |
| 1.5 Standard factorisations | 5 | |
| 1.6 Factorising quadratic trinomials | 6 | |
| 1.7 Factorising non-monic trinomials | 8 | |
| 1.8 Mixed factorisations | 10 | |
| 1.9 Algebraic fractions | 11 | |
| 1.10 Adding and subtracting algebraic fractions | 13 | |
| 1.11 Real numbers and surds | 15 | |
| 1.12 Adding and subtracting surds | 17 | |
| 1.13 The distributive law | 18 | |
| 1.14 Rationalising denominators | 19 | |
| Chapter review 1 | 22 | |
| CHAPTER 2 Trigonometry | 24 | MA-T1.1, 1.2 |
| 2.1 Review of right-angled triangles | 24 | |
| 2.2 Angles of any magnitude | 25 | |
| 2.3 Trigonometric graphs | 30 | |
| 2.4 Exact values of the trigonometric ratios | 33 | |
| 2.5 More trigonometric exact values | 35 | |
| 2.6 Direction and bearing | 38 | |
| 2.7 Angles of elevation and depression | 41 | |
| 2.8 The sine rule | 44 | |
| 2.9 The cosine rule | 49 | |
| 2.10 Area of a triangle | 53 | |
| 2.11 Applied trigonometry | 55 | |
| Chapter review 2 | 59 | |

| | | | |
|------------------|---|------------|------------------------------|
| CHAPTER 3 | Further algebraic techniques | 61 | MA-F1.1 |
| 3.1 | Linear equations in one variable | 61 | |
| 3.2 | Linear equations involving fractions | 61 | |
| 3.3 | Simple linear inequalities | 63 | |
| 3.4 | Quadratic equations. | 66 | |
| 3.5 | Quadratic equations without a linear term. | 66 | |
| 3.6 | Quadratic equations without a constant term | 67 | |
| 3.7 | General quadratic equations. | 68 | |
| 3.8 | Completing the square. | 69 | |
| 3.9 | Solving quadratic equations by completing the square. | 70 | |
| 3.10 | Quadratic equations with non-rational solutions | 71 | |
| 3.11 | Completing the square for non-monic equations | 72 | |
| 3.12 | The quadratic formula | 73 | |
| 3.13 | Problems involving quadratic equations. | 75 | |
| | Chapter review 3 | 77 | |
| | | | |
| CHAPTER 4 | Functions | 78 | MA-F1.2, 1.3, 1.4 |
| 4.1 | Functions and relations | 78 | |
| 4.2 | Sketching basic functions | 84 | |
| 4.3 | Square roots and absolute value | 88 | |
| 4.4 | Absolute value functions | 92 | |
| 4.5 | Circles. | 94 | |
| 4.6 | Cubic polynomials | 98 | |
| 4.7 | The equation $y = \frac{k}{x}$ and inverse variation | 103 | |
| 4.8 | Working with functions. | 105 | |
| | Chapter review 4. | 111 | |
| | | | |
| CHAPTER 5 | Equations and functions | 112 | MA-F1.3 |
| 5.1 | Gradient of a straight line | 112 | |
| 5.2 | Equation of a straight line | 115 | |
| 5.3 | Intersection of two lines | 119 | |
| 5.4 | Simultaneous equations. | 122 | |
| 5.5 | Problem solving with simultaneous equations. | 124 | |
| 5.6 | Solving simultaneous equations— linear and second degree. | 128 | |
| 5.7 | Solving simultaneous equations— linear and second degree in the general form | 129 | |
| 5.8 | Quadratic functions | 130 | |
| 5.9 | Parabolas and discriminants. | 133 | |

| | | |
|-------------|--|-----|
| 5.10 | Further examples involving discriminants | 136 |
| 5.11 | Solution set of simultaneous equations | 137 |
| | Chapter review 5 | 139 |

CHAPTER 6 Further trigonometry 141 MA-T1.2, 2

| | | |
|------------|---|-----|
| 6.1 | Radian measure of an angle | 141 |
| 6.2 | Arc length and sector area of a circle | 143 |
| 6.3 | Angles of any magnitude—radians | 146 |
| 6.4 | Graphs of trigonometric functions using radians | 150 |
| 6.5 | Trigonometric identities and proofs | 153 |
| 6.6 | Solving trigonometric equations | 156 |
| | Chapter review 6 | 161 |

CHAPTER 7 Introduction to differentiation 163 MA-C1

| | | |
|-------------|---|-----|
| 7.1 | Continuity and gradients of tangents | 163 |
| 7.2 | Limit and continuity | 168 |
| 7.3 | Gradient of a curve | 173 |
| 7.4 | Finding the derivative from first principles | 175 |
| 7.5 | Conditions for differentiability | 178 |
| 7.6 | Standard derivatives | 179 |
| 7.7 | The product rule | 187 |
| 7.8 | The chain rule | 189 |
| 7.9 | The quotient rule | 192 |
| 7.10 | Tangents and normals to a curve | 194 |
| 7.11 | The gradient as a rate of change | 197 |
| 7.12 | Velocity and acceleration as a rate of change | 199 |
| | Chapter review 7 | 203 |

CHAPTER 8 Exponential and logarithmic functions 205 F1.1, MA-E1

| | | |
|------------|--|-----|
| 8.1 | Index laws with integers as indices | 205 |
| 8.2 | Index laws with fractional indices | 206 |
| 8.3 | Solving equations with exponents | 208 |
| 8.4 | Logarithms | 208 |
| 8.5 | Solving equations with logarithms | 211 |
| 8.6 | Exponential functions | 215 |
| 8.7 | Some applications of exponential functions | 220 |
| 8.8 | Natural logarithms | 221 |

| | | |
|-------------|---|-----|
| 8.9 | Graphs of exponential and logarithmic functions | 223 |
| 8.10 | Logarithms in the real world. | 225 |
| | Chapter review 8 | 228 |

| | | | |
|------------------|---------------------------------------|------------|----------------|
| CHAPTER 9 | Probability | 230 | MA-S1.1 |
| 9.1 | Introduction to probability | 230 | |
| 9.2 | Venn diagrams | 237 | |
| 9.3 | Finite sample spaces. | 240 | |
| 9.4 | Successive outcomes. | 243 | |
| 9.5 | Independent events | 246 | |
| 9.6 | Dependent events. | 251 | |
| | Chapter review 9 | 257 | |

| | | | |
|-------------------|---|------------|----------------|
| CHAPTER 10 | Discrete probability distributions | 259 | MA-S1.2 |
| 10.1 | Discrete random variables. | 259 | |
| 10.2 | Expected value, variance and standard deviation of discrete probability distributions | 266 | |
| 10.3 | The uniform distribution. | 277 | |
| 10.4 | Discrete distributions in practical situations | 279 | |
| | Chapter review 10 | 283 | |

YEAR 12

| | | | |
|-------------------|--|------------|---|
| CHAPTER 11 | Descriptive statistics | 288 | SYLLABUS REFERENCE MA-S2.1 |
| 11.1 | Statistical investigations. | 288 | |
| 11.2 | Types of data | 293 | |
| 11.3 | Displaying data. | 295 | |
| 11.4 | Measures of central tendency | 308 | |
| 11.5 | Standard deviation | 321 | |
| 11.6 | Analysis of data. | 326 | |
| | Chapter review 11. | 331 | |

| | | | |
|-------------------|---|------------|--------------|
| CHAPTER 12 | Trigonometric functions and graphs | 337 | MA-T3 |
| 12.1 | Transformation of graphs of the trigonometric functions | 337 | |
| 12.2 | Further solution of trigonometric equations | 346 | |
| 12.3 | Graphical solution of equations | 350 | |
| 12.4 | Applications involving trigonometric functions and graphs | 351 | |
| | Chapter review 12. | 354 | |

| | | | |
|-------------------|--|------------|----------------|
| CHAPTER 13 | Differential calculus | 356 | MA-C2 |
| 13.1 | Approximations of trigonometric functions when x is small | 356 | |
| 13.2 | Derivatives of trigonometric functions | 358 | |
| 13.3 | Derivative of the logarithm function | 364 | |
| 13.4 | Derivative of $e^{f(x)}$ | 369 | |
| | Chapter review 13. | 371 | |
| CHAPTER 14 | The first and second derivative | 372 | MA-C3 |
| 14.1 | The sign of the derivative. | 372 | |
| 14.2 | The first derivative and turning points | 375 | |
| 14.3 | The second derivative and concavity | 379 | |
| 14.4 | The second derivative and turning points. | 383 | |
| 14.5 | Problem solving with derivatives | 390 | |
| 14.6 | Applications of the exponential and logarithmic functions | 393 | |
| 14.7 | Further applications of trigonometric functions | 396 | |
| 14.8 | Using derivatives in motion in a straight line. | 397 | |
| | Chapter review 14. | 400 | |
| CHAPTER 15 | Graphing techniques | 402 | MA-F2 |
| 15.1 | Transformation of graphs using $y = f(x + b)$ and $y = f(x) + c$ | 402 | |
| 15.2 | Transformation of graphs using $y = kf(x)$ and $y = kf(x + b)$ | 405 | |
| 15.3 | Transformation of graphs using $y = f(ax)$ and $y = f(a(x + b))$ | 407 | |
| 15.4 | Graphing rational algebraic functions. | 410 | |
| 15.5 | Applications involving graphing functions. | 412 | |
| 15.6 | Graphical solution of equations | 414 | |
| 15.7 | Regions and inequalities | 415 | |
| 15.8 | Simultaneous linear inequalities. | 419 | |
| | Chapter review 15. | 421 | |
| CHAPTER 16 | The anti-derivative | 423 | MA-C4.1 |
| 16.1 | Primitive functions. | 423 | |
| 16.2 | Indefinite integrals | 426 | |
| 16.3 | Primitives of trigonometric functions. | 428 | |
| 16.4 | Integrating the exponential function. | 429 | |
| 16.5 | Integrals resulting in logarithmic functions | 430 | |
| | Chapter review 16. | 433 | |

| | | | |
|-------------------|--|------------|----------------|
| CHAPTER 17 | Integral calculus | 434 | MA-C4.3 |
| 17.1 | Area under a curve | 434 | |
| 17.2 | The definite integral and the area under a curve | 438 | |
| 17.3 | The definite integral and the primitive function | 441 | |
| 17.4 | More areas | 448 | |
| 17.5 | Area between two curves | 450 | |
| 17.6 | Area bounded by the y -axis | 454 | |
| 17.7 | Definite integrals involving trigonometric functions | 456 | |
| 17.8 | Definite integrals involving exponential and logarithmic functions | 458 | |
| 17.9 | Applications involving integrals | 462 | |
| 17.10 | Approximate methods of integration—trapezoidal rule | 469 | |
| 17.11 | Average value of a function—an application of integration | 475 | |
| | Chapter review 17 | 477 | |
| | | | |
| CHAPTER 18 | Financial mathematics | 480 | MA-M1 |
| 18.1 | Investments and loans | 480 | |
| 18.2 | Arithmetic sequences | 490 | |
| 18.3 | Series and sigma notation (Σ) | 493 | |
| 18.4 | Arithmetic series | 495 | |
| 18.5 | Geometric sequences | 499 | |
| 18.6 | Finite geometric series | 502 | |
| 18.7 | Infinite geometric series | 505 | |
| 18.8 | Compound interest applications | 507 | |
| 18.9 | Further applications of series | 513 | |
| | Chapter review 18 | 519 | |
| | | | |
| CHAPTER 19 | Bivariate data analysis | 522 | MA-S2.2 |
| 19.1 | Types of data and their interpretation | 522 | |
| 19.2 | Scatterplots and association | 529 | |
| 19.3 | Calculating the correlation coefficient | 534 | |
| 19.4 | Modelling by finding the equation of the line of best fit | 540 | |
| 19.5 | The statistical process | 548 | |
| | Chapter review 19 | 555 | |

| | | | |
|-------------------|--|------------|--------------|
| CHAPTER 20 | Random variables | 559 | MA-S3 |
| 20.1 | Continuous probability distributions | 559 | |
| 20.2 | The normal distribution | 569 | |
| 20.3 | The standard normal distribution | 576 | |
| | Chapter review 20..... | 581 | |
| | Summary | 583 | |
| | Mathematics course outcomes | 609 | |
| | Answers | 611 | |
| | Glossary | 701 | |